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STABILITY OF COLLIDING DROPS OF IDEAL LIQUID

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The determination of the stability conditions of a system of colliding drops is of interest not only as one of the classical problems of fluid mechanics, but is urgent for the solution of a number of applied problems. A rather detailed analysis of the work on this problem performed up to 1970 is given in [1]. In the last decade interest in the physics of the interaction of drops has been stimulated by the development of the gasdynamics of two-phase flows in jets. So far, however, the laws of collision, deformation, coalescence, and disruption of drops have not been conclusively determined.

In the present article we present the results of an experimental and analytical study of the stability of a system of two colliding drops of an ideal liquid. To describe the interaction of drops quantitatively we use the following dimensionless numbers obtained by similitude theory and dimensional analysis [2, 3]:

$$\gamma = D_2/D_1 \tag{1}$$

is the ratio of the diameters of the drops, $We = \rho u^2 D_1/\sigma$ is the Weber number, $\Omega = |M|/[(D_0/2)^{7/2}\sqrt{\rho\sigma}]$ is the normalized angular momentum. Here

$$\mathbf{M} = m_1 m_2 u\delta/(m_1 + m_2), \quad D_0 = (D_1^3 + D_2^3)^{1/3}, \tag{2}$$

where m_1 and m_2 are the masses of the colliding drops, δ is the impact parameter, and M is the angular momentum of the system of drops.

Since for water drops viscous forces are negligibly small in comparison with surface tension and inertial forces, the effect of dimensionless numbers involving the viscosity (e.g., $Lp = \rho\sigma D_2/\eta^2 \sim 10^5$, $Re = \rho u D_1/\eta \sim 10^3$) is unimportant.

An experimental study of the types of interaction of water drops for We = 0.1-120 (γ = 1.9) showed that for We = 15-50 the interaction is characterized by coalescence of the drops with a subsequent possible disruption under the action of centrifugal forces [2]. Consequently, it is expedient to seek the limit of stability of a system of drops in this range of Weber numbers. We have investigated stability conditions of a system of two colliding drops for γ = 1.15-2.6 and We = 10-50. The apparatus (Fig. 1) consisted of two generators 1 producing counterstreams of water drops whose diameters could be varied from 0.3×10^{-3} to 1.2×10^{-3} m. The density, dynamic viscosity, and surface tension of the drops of distilled water were $\rho = 10^3$ kg/m³, $\eta = 10^{-3}$ kg/m·sec, and $\sigma = 73 \times 10^{-3}$ kg/sec² at 20°C. The relative velocities u of the colliding drops varied from 1 to 5 m/sec. Three-dimensional photographs were taken two SKS-1 m motion-picture cameras 2 located at right angles to one another and perpendicular to the streams of drops. Illumination was provided by photoflood lamps 3

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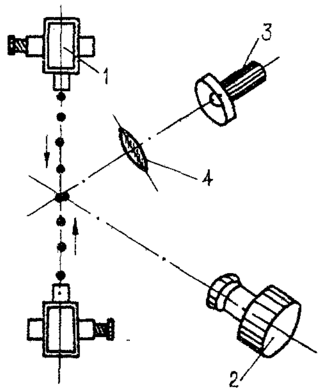

Fig. 1

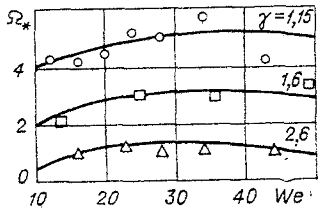

Fig. 2

with a focusing system 4 to ensure filming by the shadow method. The second motion picture camera and illumination system located symmetrically with respect to the streams of drops are not shown in Fig. 1. Photographs were taken at the rate of 1500-3000 frames per sec. The two synchronous motion picture films obtained were processed together on a decoder. The results of processing more than 300 collisions are shown in Fig. 2 as plots of Ω_* vs We for three values of γ (Ω_* is the critical value of the normalized angular momentum). The approximating curves were obtained by the least-squares method:

$$\Omega_* = 11.38 - 9.50\gamma + 0.13\,\mathrm{We} - 0.88\cdot10^{-2}\gamma\,\mathrm{We} - 0.16\cdot10^{-2}\mathrm{We}^2 + 1.90\gamma^2.$$

If for fixed γ and We the values of Ω lie above the Ω_* curve, the system of drops is unstable and breaks up after interaction; if the values of Ω fall in the stable region below the Ω_* curve, the drops coalesce.

An analytical estimate of the critical levels in the collisions of drops can be obtained within the framework of the model of an ideal liquid. We seek the dependence of Ω_* on γ only, assuming that in the type of interaction considered, which is characteristic for We = 10-50 [2], the dependence of Ω_* on We is unimportant. The validity of this assumption is confirmed by the experimental results shown in Fig. 2. We consider a simplified model of a grazing collision in which the impact parameter $\delta \simeq D_1/2 + D_2/2$. Since the required relation $\Omega_*(\gamma)$ does not contain δ explicitly, it will hold also for collisions which are not grazing. If the angular momentum Ω is less than critical in a grazing collision, one should expect the drops to coalesce; the system of coalesced drops will revolve, and their centers of mass will converge. If $\Omega > \Omega_*$ the drops separate from one another after touching surfaces. For $\Omega = \Omega_*$ the drops revolve about the center of mass of the system, not separating and not coalescing.

The equilibrium conditions in the last case can be found from the minimum potential energy variational principle [4] by equating to zero the first variation of the potential energy of the system in a coordinate system rotating with constant angular velocity ω. The expression for the potenital energy of the system of drops has the form [4]

$$E = -(1/2)I\omega^2 + \sigma S, \tag{3}$$

where I is the moment of inertia of the system.

We calculate the area S approximately by assuming it is equal to the sum of the surface S_1 and S_2 of the two drops minus twice the area of contact. Then, to within quantities of the order h^2 (Fig. 3), we obtain

$$S = S_1 + S_2 - 2\pi h D_1 D_2/(D_1 + D_2), \tag{4}$$

where $h = h_1 + h_2$ is the depth of penetration.

Since the effect of viscosity is not taken into account in the interaction, in the revolution of the drops under equilibrium conditions we consider only the revolution of their centers of mass about the center of mass of the system. We neglect the rotation of each

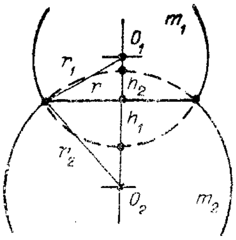

Fig. 3

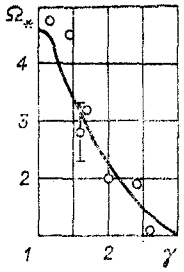

Fig. 4

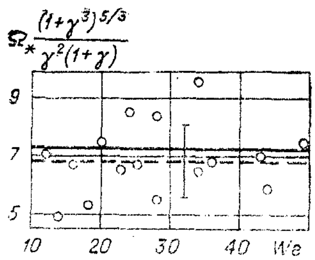

Fig. 5

drop around its own center of mass. In this approximation the moment of inertia of the system with respect to its center of mass is

$$I = \frac{m_1 m_2}{m_1 + m_2}\left(\frac{D_1}{2} + \frac{D_2}{2} - h\right)^2. \tag{5}$$

Substituting Eqs. (4) and (5) into (3), we obtain an expression for the potential energy in the rotating coordinate system:

$$E = -\frac{1}{2}\omega^2 \frac{m_1 m_2}{m_1 + m_2}\left(\frac{D_1}{2} + \frac{D_2}{2} - h\right)^2 + \sigma(S_1 + S_2) - 2\pi h \frac{D_1 D_2}{D_1 + D_2}.$$

The system of drops is in equilibrium if in the neighborhood of h = 0 for ω = const

$$\delta E = \omega^2 \frac{m_1 m_2}{m_1 + m_2}\left(\frac{D_1}{2} + \frac{D_2}{2} - h\right) - 2\pi\sigma \frac{D_1 D_2}{D_1 + D_2} = 0. \tag{6}$$

Substituting (1) and (2) and the relation **M** = Iω into (6), we obtain

$$\Omega_* \frac{(1+\gamma^3)^{5/3}}{\gamma^2(1+\gamma)} = \frac{4\pi}{\sqrt{3}} = 7.26. \tag{7}$$

The curve in Fig. 4 shows Ω_* as a function of γ calculated from Eq. (7). The points are experimental values of $\Omega_*(\gamma)$ averaged over We. The experimental points for the complex on the left-hand side of Eq. (7) are shown in Fig. 5 as a function of We. Considering the spread of the experimental data, these points lie around the value 6.84 ± 1.3, which is in satisfactory agreement with the analytical estimate.

Within the ranges We = 10-50 and γ = 1.15-2.6 investigated, Eq. (7) can be recommended for estimating the limits of stability of a system of two colliding drops of an ideal liquid.

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